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What is an Optimal Spacecraft Structure?

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Many engineers have attempted to optimize the design of a spacecraft structure. They usually utilize a simplified criterion or objective function (e.g., fully stressed, minimum cost) that addresses only one aspect of the design. The real-world dilemma is closer to having to select among competing designs using multiple criteria of varying importance as the time or emphasis changes. This paper presents a method for evaluating candidate designs that allows the user to assign weights to the various criteria. These weights can be altered as the program objectives change. The method makes use of the experts from the numerous disciplines involved in the design, analysis, manufacture, and operation of a spacecraft.

Nomenclature

B_j = evaluation parameter
$F(x)$ = objective function
$G(x)$ = inequality constraints
$H(x)$ = equality constraints
S_j = score for item j
S_{total} = total score for a particular design using a prescribed set of weighting factors
T_j = target for weight or cost (item j)
V_j = value of weight or cost
W_j = weighting factor for item j
$\boldsymbol{x}^i$ = vector of design variables
x_L = vector of design variable lower limit
x_U = vector of design variable upper limit
Y_i = search direction vector
α = distance to move in the design space

Introduction

MANY papers and books have been written on structural optimization.[1-11] Most of the methods use such parameters as lowest material cost, lowest weight, or fully stressed design as part or all of their objective function or design criteria in the selection of a structural arrangement.

This paper addresses other issues that are becoming more relevant in the design of today's spacecraft structures: 1) ease and cost of manufacturing and inspection, 2) material cost and availability, 3) test and analysis considerations, 4) expanded mission requirements, and 5) dynamic characteristics. The paper first reviews the classical methods of structural optimization, then recommends a new way to evaluate spacecraft structural designs. The new method looks at a spacecraft structure from operations, producibility, test requirements, and other points of view in addition to considering weight and cost. Examples of the application of this method are given.

Historical Development

The history of structural optimization dates back to the mid-1600s, when Galileo sought to find the optimal shape of a variable-depth beam. As structural analysis progressed, so did attempts to minimize weight. Bernoulli, making the assumption that plane sections remain plane, used differential calculus to design a beam of uniform strength. Other researchers, including Newton, Parent, Lagrange, Clausen, Levy, and Maxwell, developed theories and theorems advancing structural analysis and design. In the mid-1900s, minimum-weight structural design became more important with the development of aircraft (especially during the Second World War). With buckling considered as the primary constraint on aircraft structures, Wagner, Smith, Cox, Zahorski, Wittrick, and others examined the minimum-weight designs for simple panels, stringer-stiffened panels, and sandwich panels.

Today, structural optimization is advancing rapidly in concert with advances in digital computing and numerical methods; however, it also must deal with the problems of multiple-objective functions, constraints from multiple disciplines, and a large number of design variables. Optimization methods are used on an assortment of problems, such as composite laminate design, vehicle shape design, and the tuning of an analysis model to match test data.

Optimization techniques provide designers and analysts the potential to significantly reduce engineering time and costs and yield improved designs or to locate a feasible solution. However, it is necessary to understand that optimization is only a design tool and will rarely provide the absolute best design.

In short, structural optimization provides the following: 1) a systematic design process requiring minimum human intervention, 2) reduced design time, 3) the ability to deal with many design variables and constraints concurrently, 4) design improvement, and 5) feasible solutions to complex design problems. Optimization has the following limitations:

1) With the exception of unimodal problems, optimization cannot guarantee a global optimal unless an exhaustive search is done (which may require a prohibitive amount of CPU time).

2) If the analysis program is not theoretically precise or if the design problem formulation is not accurately and adequately defined, the optimization results may be misleading.

3) Optimization will invariably utilize errors to yield mathematical design improvements.

Structural optimization has two fundamental approaches: 1) optimality criterion methods, and 2) numerical optimization methods. The optimality criterion methods' objective is to obtain a design that satisfies a selected criterion that it is believed will minimize the weight of the structure. The selected criterion is generally intuitive but can also be mathematically defined. Optimality criterion methods, like numerical optimization methods, are iterative solutions and, also like numerical optimization methods, impose constraints on the structure, such as allowable stress or maximum displacements. The process is fundamentally a two-step procedure: 1) structural analysis, and 2) structural redesign. The analysis is typically done using finite elements, and a recurrence relation is used to redesign. The redesign changes the design variables in such a

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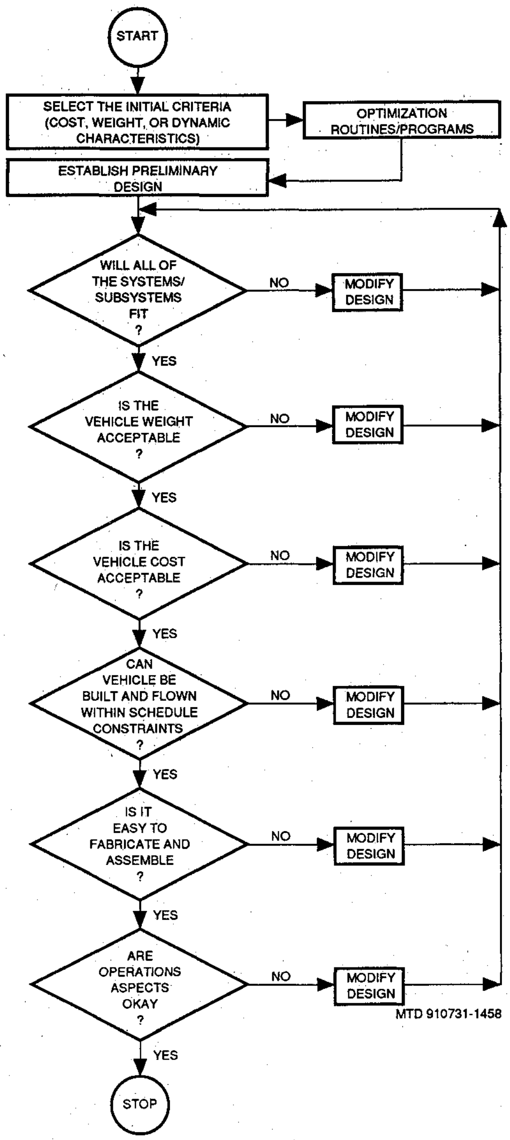

Fig. 1 Typical structural design (optimization) approach.

way that the structure approaches the optimality criterion. Depending on the problem and the recurrence relationship, the process may not converge, in which case a new approach would be needed.

Numerical structural optimization uses nonlinear mathematical programming techniques to locate a minimum in a feasible design space. It is concerned with achieving a structural design that minimizes an objective function, usually weight, while also satisfying system constraints, such as stress or deflection limitations. Mathematically, the structural optimization problem is stated as follows. Minimize $F(x)$, the objective, such that $H(x)=0$, the equality constraints, and $G(x)\neq 0$, the inequality constraints, and $x_L > x > x_U$, the side constraints.

The optimization problem is nonlinear when at least one of the previous equations is nonlinear. Typically, the structural optimization problem is a nonlinear problem whose solution requires an iterative process that involves five major steps: 1) development of an initial design, 2) analysis of the design, 3) sensitivity analysis, 4) redesign, and 5) assessment of design convergence (return to step 2 if necessary).

When the optimization problem has multiple minima, the development of the initial design can have a significant influence on the final optimal design configuration found. This dependence on the initial design for a solution is typical of numerical approaches for the analysis of nonlinear equations. The challenge is to find the initial design that yields the global minimum.

The initial design also affects the number of solution search cycles required to converge on an optimal design. In fact, with the selection of some initial designs, the process may fail to converge.

Numerical structural analysis, step 2, is an important part of the systematic design process because it verifies that the design is within the feasible region (i.e., no constraints have been violated). Finite element analysis technology, the dominant method used in numerical structural analysis, enables the designer to reliably and efficiently analyze highly complicated structures as accurately as he/she desires. Indeed, the development of systematic procedures for optimizing structural configurations is a natural extension of the development of numerical analysis procedures.

The evaluation of constraint function derivatives with respect to design variables constitutes design sensitivity analysis. The calculation of these derivatives poses a serious obstacle to the application of structural optimization to configurations with many design variables and constraints because of the high computational costs associated with implementation.[11]

Three methods for computing sensitivity can be found in the literature: 1) finite differencing, 2) analytic differentiation, and 3) the energy sensitivity approach.[12,13] Analytic differentiation is generally much more cost effective than finite differencing. The energy sensitivity approach is a new approach that uses an intrinsic, symmetric, positive definite or positive semidefinite matrix to evaluate sensitivities. For the analytic differentiation approach, g is the response vector (e.g., stress,

Table 1 Initial weighting factors used in objective function

Item	Factor
Weight	0.20
Cost (DDT&E)	0.20
Cost (recurring)	0.20
Ease of fabrication	0.05
Ease of system/subsystem installation	0.05
Ease of inspection/repair	0.05
Availability of material	0.05
Build time	0.05
Test requirements	0.05
Analysis techniques available	0.05
Operations schedule	0.05

Table 2 Examples of cost/weight ratings

Item j	Item name	Value V_j	Target T_j	Percent over or under	Score S_j
1	Weight	160,000 lb	150	0.067 over	8.67
1	Weight	145,000 lb	150	0.033 under	10.66
2	Cost (DDT&E)	$120 million	110	0.091 over	8.18
3	Cost (recurring)	$10 million	15	0.33 under	16.67

Table 3 Sample weighting factors

Item	Weighting factor conditions A	B	C	D	E
Weight	0.20	0.40	0.10	0.20	0.10
Cost (DDT&E)	0.20	0.10	0.10	0.30	0.20
Cost (recurring)	0.20	0.10	0.40	0.10	0.30
Ease of fabrication	0.05	0.05	0.05	0.10	0.05
Ease of system/subsystem installation	0.05	0.05	0.04	0.02	0.05
Ease of insepction/repair	0.05	0.05	0.03	0.02	0.05
Availability of material	0.05	0.05	0.03	0.02	0.05
Build time	0.05	0.10	0.10	0.10	0.05
Test requirements	0.05	0.05	0.00	0.10	0.05
Analysis techniques available	0.05	0.00	0.05	0.02	0.05
Operations schedule	0.05	0.05	0.10	0.02	0.05

displacement, etc.) K the stiffness matrix, U a matrix of column vectors of unknown nodal displacements, and P a matrix of column vectors of nodal loads.

The sensitivity matrix is expressed as

$$S=\frac{\mathrm{d}g}{\mathrm{d}x}=\left(\frac{\partial g}{\partial x}\right)^T+\left(\frac{\partial g}{\partial U}\right)^T K^{-1}\left[\left(\frac{\partial P}{x}\right)-\left(\frac{\partial K}{\partial x}\right)U\right] \quad (1)$$

where the second term on the right requires significant computational effort. The numeric evaluation of this term is accomplished using either the direct approach (also called design variable or behavioral space approach) or the adjoint approach (also called the response space approach). Both the adjoint and direct approaches use Eq. (1), but it is the order in which the matrix operations are performed that distinguishes the two. The advantage of one approach over the other is problem dependent. In the direct approach, the number of calculations is proportional to the number of design variables times the number of load vectors. In the adjoint approach, the calculations are proportional to the number of design constraints (e.g., stress, displacement, etc.).

The fourth step of the optimization process is to synthesize a better design—one that improves the system's objective function. Using the appropriate optimizer is important since it will affect how many iterations of the overall design cycle are run before an optimum is reached. Probably the most common form of the redesign process is

$$X^i=X^{i-1}+\alpha Y^i \quad (2)$$

where i is the iteration number, Y a search direction vector, and α a scalar that defines the distance to move in direction S in the design space.

The final step of the iterative process assesses the convergence of the design. The convergence criteria might be any or all of several popular tests, such as the Kuhn-Tucker conditions, changes in objective function gradients, or simply a limit on the number of iterations.

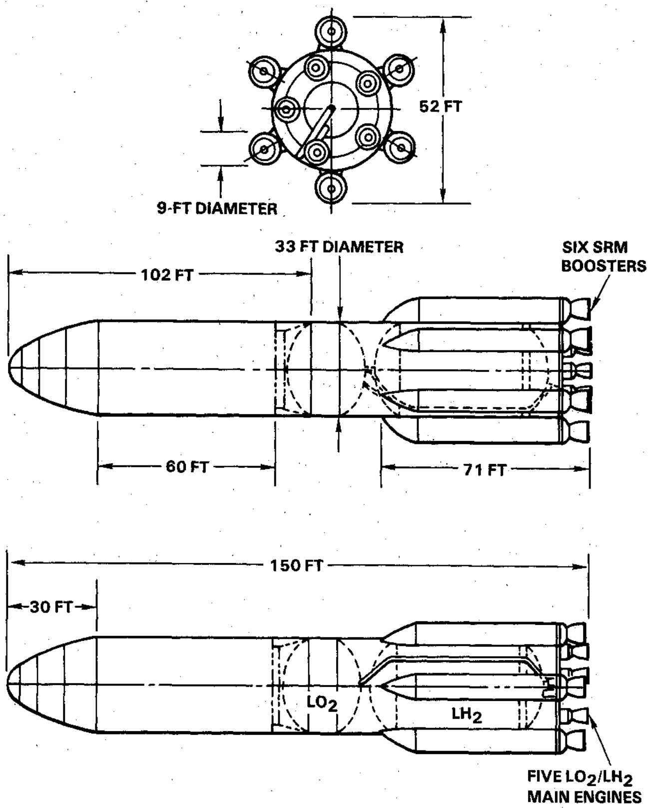

Fig. 2 Spacecraft design (example 1).

Table 4 Description of weighting factor conditions

Weighting factor condition	Description
A	Standard initial factors
B	Low weight, short build time
C	Low recurring cost, short build time
D	Low development cost, low weight
E	Low recurring cost, low DDT&E cost

Table 5 Scores for example problems

		Scores, S_j	
J	Description	Example 1	Example 2
1	Weight	8.0	9.0
2	Cost (DDT&E)	9.0	9.5
3	Cost (recurring)	8.0	8.5
4	Ease of fabrication	9.5	8.5
5	Ease of system/subsystem installation	9.5	9.5
6	Ease of inspection/repair	8.5	9.5
7	Availability of material	9.0	9.0
8	Build time	9.0	8.0
9	Test requirements	9.0	8.5
10	Analysis techniques available	10.0	8.0
11	Operations schedule	8.0	7.5

Table 6 Total scores (overall evaluations) of candidate designs

Weight factor condition	Description	Total score Example 1	Example 2
A	Standard initial factors	86.4	88.5
B	Low weight, short build time	80.7	88.5
C	Low recurring cost, low weight	84.7	85.7
D	Low DDT&E cost, low weight	87.5	88.7
E	Low recurring cost, low DDT&E	82.2	70.6

Current Spacecraft Structural Design

There are virtually no spacecraft flying today that were completely designed by using structural optimization techniques. Some have been significantly influenced by these methods, but usually only a few design aspects (e.g., weight, dynamic characteristics) are involved. A spacecraft may start off with a preliminary design based on optimization methods, but the design is usually modified for other important reasons. A typical design flow is shown in Fig. 1. When the design is modified to account for subsystem installation, the build schedule, and operational and other considerations, the total structural system is not usually completely reassessed (i.e., how far it is from minimum weight). This means that a design could start close to minimum weight but would no longer be close after it is modified to satisfy other requirements.

When an optimization program is used in the preliminary design of a spacecraft, it is usually used in conjunction with a particular load condition or a small group of load conditions. The definition of loads for a spacecraft is an iterative process. The flexible-body loads are a function of the vehicle's dynamic characteristics (mode shapes and frequencies), weight, center of gravity, shape/aerodynamic characteristics, trajectory, and a few other items. Most of these change as the design is modified. Therefore, the loads change as the design changes.

Suggested Method for Evaluation of Structural Designs

Structural engineers typically attack the issue of spacecraft design from the following perspectives: 1) weight, 2) cost (primary structure), and 3) ease of fabrication and assembly. However, when a spacecraft is actually being manufactured and flown, the list of important issues is closer to 1) weight;

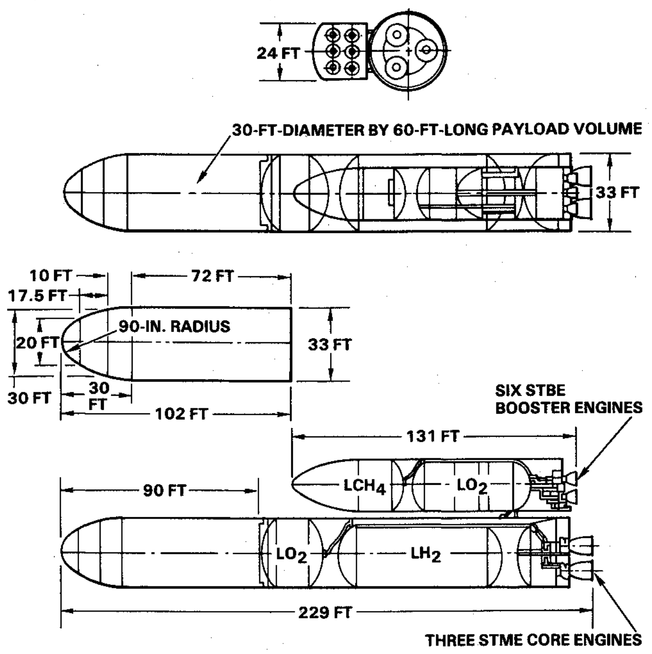

Fig. 3 Spacecraft design (example 2).

2) design, development, test, and evaluation (DDT&E) cost;; 3) recurring cost; 4) ease of fabrication and assembly; 5) ease of system and subsystem installation; 6) ease of inspection and repair; 7) availability of material; 8) build time; 9) test requirements; 10) analysis techniques available; and 11) operations schedule. All of these can be included in the design evaluation process. An objective function has been developed that incorporates these 11 items.

Objective Function

The objective function provides a way to simultaneously consider many aspects of the design. Weighting factors (W_j) are used in conjunction with item scores (S_j) to provide an overall evaluation of a candidate design. An initial set of weighting factors is suggested in Table 1. Before a design can be evaluated using these factors each item must receive a score ($S_j, j=1,\ldots,11$).

For the first three items, a qualitative score can be established on the basis of some percentage above or below a target value (T_j). Mathematically, this is expressed as

$$S_j = (10) - (B_j)\left(1 - \left[\frac{V_j}{T_j}\right]\right) \qquad \text{for} \quad (V_j \geq T_j) \tag{3}$$

or

$$S_j = (10) + (B_j)\left(1 - \left[\frac{V_j}{T_j}\right]\right) \qquad \text{for} \quad (T_j \geq V_j > 0) \tag{4}$$

Typically, $B_j = 20$ so that a value (V_j) 5% over the target (T_j) causes a one-point penalty ($S_j = 9$). A set of example values is given in Table 2.

The user obtains a rating value for each of the other items on the list; that is, S_1 through S_{11} are the subjective evaluations of the experts in those respective disciplines. These values can be any number between 0.0 and 10.0. These values (S_j), along with the weighting factors W_j, are used in the objective function S_{total} computations

$$S_{\text{total}} = \sum_{j=1}^{11} (W_j)(S_j) \tag{5}$$

where

$$\sum_{j=1}^{11} W_j = 1.00 \tag{6}$$

This method allows the user to evaluate a design in as many different ways as he or she desires for very little cost. For a particular project, each of the following disciplines or individuals might construct its own set of weighting factors: 1) program manager 2) chief engineer, 3) structural designer, 4) structural analyst, 5) manufacturing, 6) testing, and 7) operations. The given candidate designs would be evaluated using the different sets of weighting factors. Tables 3 and 4 present five different sets and a brief description of each.

Application

This section shows how the method is applied to a spacecraft design. Two spacecraft designs are shown in Figs. 2 and 3. The 11 scores, S_I through S_{11}, were determined by the appropriate experts. A complete listing of these values is given in Table 5.

Each of the designs was evaluated using the five different sets of weighting factors. The total scores are summarized in Table 6. This technique readily lends itself to the performance of many types of sensitivity studies. The total scores were obtained using

$$S_{\text{total}} = \sum_{j=1}^{11} (W_j)(S_j) \tag{7}$$

The total score S_{total} is an indicator of how well the design fulfills the desires associated with a given set of weighting factors.

Conclusions

A new method of evaluating spacecraft structural designs has been presented. This method considers and evaluates a design from numerous points of view simultaneously. It allows a program manager or chief engineer to make an objective judgement using predetermined weighting factors or to perform sensitivity studies by merely modifying the weighting factors.

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